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Altmann, Klaus (D-HUMBM2); **van Straten, Duco** (D-MNZ)
The polyhedral Hodge number $h^{2,1}$ and vanishing of obstructions. (English summary)
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To any compact convex polytope $\Delta \subset \mathbf{R}^n$ (defined over a subfield $K \subset \mathbf{R}$) the authors associate a certain cohomological system. It is defined in terms of the non-complete fan given by a cone over Δ. Such a cohomological system is a covariant functor from the mentioned fan (viewed as a category in a natural way) to the category of abelian groups. Namely, to a cone in the mentioned fan one associates the residue vector space modulo its affine hull. This cohomological system gives rise to cohomology groups in a standard way and the obtained groups $D^k(\Delta)$ are essentially Brion's groups $H^{k,1}$, associated to the normal fan of Δ [M. Brion, Tohoku Math. J. (2) **49** (1997), no. 1, 1–32; MR1431267 (98a:52019)]. But the new interpretation provides a possibility of proving new vanishing results, especially for $D^2(\Delta)$ under some combinatorial restrictions on Δ. This is a consequence of the description of $D^2(\Delta)$ (when $D^1(M) = D^2(M) = 0$ for every 3-face $M \subset \Delta$) as a K-vector subspace in the appropriate bigger space by explicit linear equations. These equations are derived by a thorough study of the relevant differential in a spectral sequence, associated to the covering by subfans.

In the concluding section the authors relate the invariants $D^k(\Delta)$ with the cotangent cohomology modules $T^k(X_{\mathrm{cone}(\Delta)})$ of the Gorenstein toric variety associated with a lattice polytope Δ. In certain cases these groups coincide. In this situation the $T^k(X_{\mathrm{cone}(\Delta)})$ are independent of the interaction of Δ with the lattice structure of the ambient space because the $D^k(\Delta)$ only depend on the projective equivalence class of Δ. If $D^2(\Delta) = T^2(X_{\mathrm{cone}(\Delta)})$, then the vanishing of $D^2(\Delta)$ has a deformation-theoretic meaning—i.e., the deformations are unobstructed.

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.